Suppose you are trying to find all $\theta \in [0, 2\pi)$ such that

trigfunction $\theta = c$ (where c is in the range of *trigfunction*)

where *trigfunction* is either $\sin$, $\cos$ or $\tan$, and θ is clearly not one of the usual multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ or π.
(The logic below also works if θ is one of the usual multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ or π.)

NOTE: If *trigfunction* is either $\csc$, $\sec$ or $\cot$, immediately rewrite the equation as 1 / *trigfunction* $\theta = 1/c$.

NOTE: The arcsine or arctangent of a negative number is in the interval $[-\frac{\pi}{2}, 0]$ or $(-\frac{\pi}{2}, 0]$ respectively.
The arccosine of a negative number is in the interval $[\frac{\pi}{2}, \pi]$.

[1] Let θ_{REF} = *arc-trigfunction* $|c|$, the reference angle for your solutions.
(Simplify *arc-trigfunction* $|c|$ if possible – see Math 1C Prerequisites Review handout question [7].)
The reference angle will be in quadrant 1, and $\theta_{REF} \in [0, \frac{\pi}{2}]$.

[2] Determine in which quadrants *trigfunction* would have the same sign as c.

[3] Draw diagrams corresponding to angles in each of the quadrants in [2].
The **acute** angle between the angle you're trying to find and the $x-$axis is θ_{REF}.

[4] Determine the measure of the angle (measured counterclockwise starting from the polar / positive $x-$axis) you're trying to find in terms of θ_{REF}, then replace θ_{REF} with *arc-trigfunction* $|c|$.

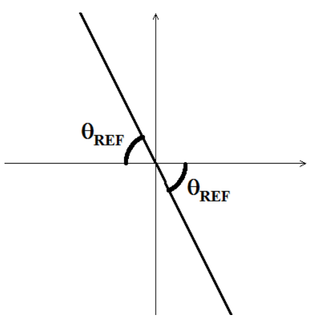
EXAMPLE

Solve $\tan\theta = -2$ where $\theta \in [0, 2\pi)$. NOTE: -2 is in the range of $\tan\theta$, which is $(-\infty, \infty)$, so θ exists.

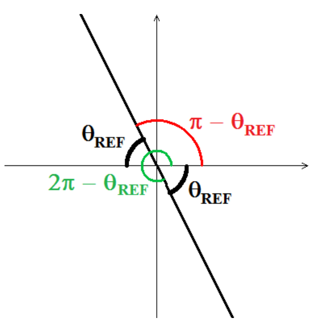
[1] $\theta_{REF} = \arctan|-2| = \arctan 2$

[2] $\tan\theta$ is negative (like -2) in quadrants 2 and 4

[3]



[4]



$\theta = \pi - \arctan 2$ or $2\pi - \arctan 2$

NOTE: The θ in quadrant 4 is coterminal with $\arctan(-2)$, but $\arctan(-2) \in (-\frac{\pi}{2}, 0]$.
ie. $\arctan(-2) \notin [0, 2\pi)$ as required. So, $\theta \neq \arctan(-2)$.

NOTE: The θ in quadrant 2 is not $\arctan(-2)$ because the range of $\arctan x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$, which only includes certain angles in quadrants 1 and 4.

Now, try these

[a]	$\sin\theta = \frac{1}{4}$	[b]	$\cos\theta = \frac{1}{4}$	[c]	$\tan\theta = \frac{1}{4}$
[d]	$\sin\theta = -\frac{1}{4}$	[e]	$\cos\theta = -\frac{1}{4}$	[f]	$\tan\theta = -\frac{1}{4}$
[g]	$\sec\theta = 3$	[h]	$\csc\theta = 3$	[j]	$\cot\theta = 3$
[k]	$\sec\theta = -3$	[m]	$\csc\theta = -3$	[n]	$\cot\theta = -3$

where $\theta \in [0, 2\pi)$ for all questions.

Now, suppose you are trying to find all $\theta \in [0, 2\pi)$ such that

trigfunction $n\theta = c$ (where c is in the range of *trigfunction*, and n is a positive integer)

[A] Replace $n\theta$ with t and find all $t \in [0, 2\pi)$ such that *trigfunction* $t = c$ using the process above.
Suppose the solutions are $t = t_1$ or t_2.
NOTE: If $c = -1$ or 1, and *trigfunction* is either $\sin$ or $\cos$, there will only be 1 value of t, not 2.

[B] Since $0 \le \theta < 2\pi$, therefore $0 \le n\theta < 2n\pi$.
So, in order to find all values of $n\theta$, you need to go around the unit circle n times and find all angles coterminal with t_1 and t_2.
So, $n\theta = t_1$ or t_2 or $2\pi + t_1$ or $2\pi + t_2$ or $4\pi + t_1$ or $4\pi + t_2$ or … or $2(n-1)\pi + t_1$ or $2(n-1)\pi + t_2$.

[C] Divide both sides by n to get

$$\theta = \frac{t_1}{n} \text{ or } \frac{t_2}{n} \text{ or } \frac{2\pi + t_1}{n} \text{ or } \frac{2\pi + t_2}{n} \text{ or } \frac{4\pi + t_1}{n} \text{ or } \frac{4\pi + t_2}{n} \text{ or } \ldots \text{ or } \frac{2(n-1)\pi + t_1}{n} \text{ or } \frac{2(n-1)\pi + t_2}{n}.$$

EXAMPLE

Solve $\sin 3\theta = -\frac{1}{4}$ where $\theta \in [0, 2\pi)$.

[A] From question [d] above, $\sin t = -\frac{1}{4}$ where $t \in [0, 2\pi)$ has solutions $t = \pi + \arcsin\frac{1}{4}$ or $2\pi - \arcsin\frac{1}{4}$

[B] Since $0 \le \theta < 2\pi$, therefore $0 \le 3\theta < 6\pi$ (3 times around the unit circle).

So, $3\theta = \pi + \arcsin\frac{1}{4}$ or $2\pi - \arcsin\frac{1}{4}$ (1^{st} time around the unit circle)
or $2\pi + \pi + \arcsin\frac{1}{4}$ or $2\pi + 2\pi - \arcsin\frac{1}{4}$ (2^{nd} time around the unit circle)
or $4\pi + \pi + \arcsin\frac{1}{4}$ or $4\pi + 2\pi - \arcsin\frac{1}{4}$ (3^{rd} time around the unit circle)

ie. $3\theta = \pi + \arcsin\frac{1}{4}$ or $2\pi - \arcsin\frac{1}{4}$ or $3\pi + \arcsin\frac{1}{4}$ or $4\pi - \arcsin\frac{1}{4}$ or $5\pi + \arcsin\frac{1}{4}$ or $6\pi - \arcsin\frac{1}{4}$

[C] So, $\theta = \frac{\pi}{3} + \frac{1}{3}\arcsin\frac{1}{4}$ or $\frac{2\pi}{3} - \frac{1}{3}\arcsin\frac{1}{4}$ or $\pi + \frac{1}{3}\arcsin\frac{1}{4}$ or $\frac{4\pi}{3} - \frac{1}{3}\arcsin\frac{1}{4}$ or $\frac{5\pi}{3} + \frac{1}{3}\arcsin\frac{1}{4}$ or $2\pi - \frac{1}{3}\arcsin\frac{1}{4}$

Now, try these

[i]	$\sin 2\theta = \frac{1}{4}$	[ii]	$\cos 3\theta = \frac{1}{4}$	[iii]	$\tan 5\theta = \frac{1}{4}$
[iv]	$\sin 3\theta = -\frac{1}{4}$	[v]	$\cos 5\theta = -\frac{1}{4}$	[vi]	$\tan 3\theta = -\frac{1}{4}$
[vii]	$\sec 2\theta = 3$	[viii]	$\csc 4\theta = 3$	[ix]	$\cot 4\theta = 3$
[x]	$\sec 4\theta = -3$	[xi]	$\csc 5\theta = -3$	[xii]	$\cot 2\theta = -3$

where $\theta \in [0, 2\pi)$ for all questions.